1.0 Introduction

This investigation aims to determine the mathematical relationship between the angle of elevation of a ramp and the average speed of a tennis ball rolling down it. The context for this task is provided by a sports brand, Tennis Solutions, which requires the accurate calibration of a tennis ball launcher to ensure it delivers balls at a consistent speed of 1.63 m/s. To calibrate the launcher, this investigation explores the motion of a tennis ball an inclined ramp. The mathematical focus of this task is the development of a linear regression model describing the relationship between ramp angle (independent variable) and average speed (dependent variable). The key mathematical concepts integrated in this investigation:

- Kinematics: motion of a rolling object on an inclined plane
- Trigonometry: determining angles from vertical height and ramp length
- Regression fitting a straight-line model to experimental data

The model will then be applied to predict the angle of elevation and vertical height required for the ball to reach target speed of 1.63 m/s.

2.0 Results and Discussion

2.1 Observations and Assumptions

Several assumptions were made during this investigation to simplify the problems and ensure that a clear mathematical model could be developed:

- Negligible friction and air resistance: The ramp surface was assumed to be smooth; flat and air resistance was not considered. This assumption allowed for a simple model based only on gravity and angle.
- Consistent rolling of tennis ball: It was assumed that the tennis ball was rolled without slipping and without wobbling or bouncing.
- Constant ramp length: Ramp length is fixed at 1.20 m which unsured that only the vertical height changed and thus only the angled varied.
- Manual timing with stopwatch: Human reaction time could introduce small inconsistencies. To reduce inaccurate data, three trails were conducted for each ramp height and average time was used.

These assumptions were necessary to construct a simple mathematical model, but they also heavily impacted the accuracy of the final solution.

During the experiment:

- The tennis ball accelerated steadily down the ramp without visible bouncing.
- Larger vertical heights produced noticeably shorter travel times.
- Timings variations between trials were small but present, consistent with human reaction delays.

These observations suggest that the relationship between ramp angle and speed is increasing and approximately linear within the tested range.

2.2 Data Collection and Processing

Variables Defined:

- Θ = angle of elevation of ramp (°) *(independent variable)*
- t = average time taken for ball to travel the ramp (s)
- d = length of ramp = 1.20 m *(constant)*
- v = average speed of the ball (m/s) *(dependent variable)*
- h = vertical height of ramp (cm)

The experiment involved rolling a standard tennis ball down a ramp of constant length of 120 cm. This was done in various vertical lengths to change the angle of elevation. For each height, the time it took for the tennis ballot reach the bottom was recorded over three trails to improve accuracy. The average speed was calculated using:

$$Average\ Speed\ = \frac{Ramp\ Length}{Average\ Time}$$

Raw Data Table

Angle (°)	Height (cm)	Trial 1 (s)	Trial 2 (s)	Trial 3 (s)	Average time (s)	Average Speed (cm/s)	Average speed (m/s)
55	100	0.61	0.62	0.61	0.61	196.72	1.9672
50	95	0.65	0.59	0.63	0.62	193.55	1.9355
48.59	90	0.63	0.7	0.66	0.66	181.81	1.8181
47.46	85	0.7	0.76	0.77	0.74	159.01	1.5901
42.73	80	0.77	0.74	0.77	0.76	157.89	1.5789
38.68	75	0.71	0.75	0.8	0.75	150	1.5
35.75	70	0.87	0.87	0.82	0.85	141.18	1.4118
33.21	65	0.9	0.88	0.9	0.89	134.83	1.3483
30.51	60	0.91	1	0.97	0.96	125	1.25
28.07	55	0.93	0.95	0.93	0.94	123.71	1.2371
26.05	50	1.07	1.1	0.98	1.05	114.22	1.1422
23.96	45	1.06	1.02	1.05	1.04	115.03	1.1503
21.8	40	1.1	1.13	1.19	1.15	104.35	1.0435
19.47	35	1.27	1.3	1.34	1.3	92.07	0.9207
17.46	30	1.38	1.3	1.36	1.35	88.06	0.8806
14.93	25	1.44	1.45	1.54	1.48	81.08	0.8108
9.59	20	1.72	1.76	1.73	1.74	63.06	0.6306
7.18	15	2.14	2.02	2.09	2.08	57.69	0.5769
4.78	10	2.92	2.94	3.03	2.96	40.54	0.4054

Processed Data Table

Angle (°)	Speed (m/s)

5.2	0.55
6.5	0.59
8	0.63
9.7	0.68
11.4	0.74
13.1	0.78
14.9	0.84
16.6	0.87
18.4	0.92

The angle of elevation for each height was calculated using the trigonometric identity:

Where **h** is the vertical height in cm and 120 is the ramp length is cm.

$$\sin sin\,\theta\ =\ \frac{Height}{Ramp\ Length}\ \therefore\theta\ =\ \sin sin\ -\ 1\ \left(\frac{h}{120}\right)$$

Example calculation: If the vertical height was 11 cm:

$$-\ 1\left(\frac{11}{120}\right) = \sin sin\ -\ 1\ (0.0917)\approx 5.2°$$

The average speed of the tennis ball was determined by:

$$v\ =\ \frac{d}{t}$$

Example calculation: If the average time was 2.18 s, then:

$$v\ =\ \frac{1.20}{2.18} = 0.55\ m/s$$

Where **d** = 1.20 m is the ramp length and **t** are the average time in seconds.

2.3 Drawing the Line of Best Fit by Hand

To calculate the equation of the line of best fit manually, two points on the drawn line were selected:

Point A: $\left(x_1,\ y_1\right) = (5.2, 0.55)$

Point B: $\left(x_2 y_2\right) = (18.4, 0.92)$

Step 1: Calculate the Gradient

$$m\ =\ \frac{y_2 - y_1}{x_2 - x_1}$$

$$m\ =\ \frac{0.92-0.55}{18.4-5.2} = \frac{0.37}{13.2} = 0.0280$$

Step 2: Calculate the y-intercept

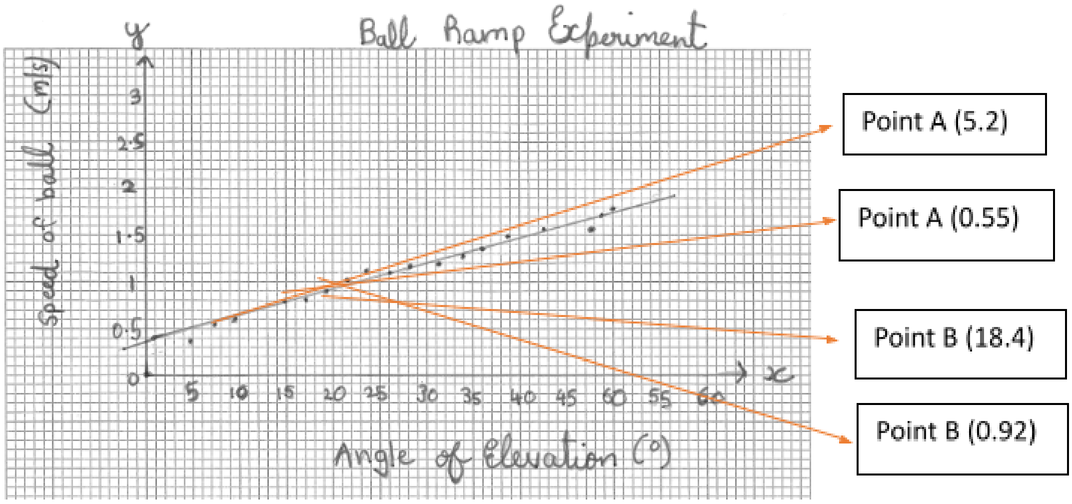
Using $y = mx + c$ and substituting $(5.2, 0.55)$:

$$0.55 = 0.0280(5.2) + c \therefore c = 0.55 - 0.1456 \approx 0.404$$

Step 3: Write the equation of line

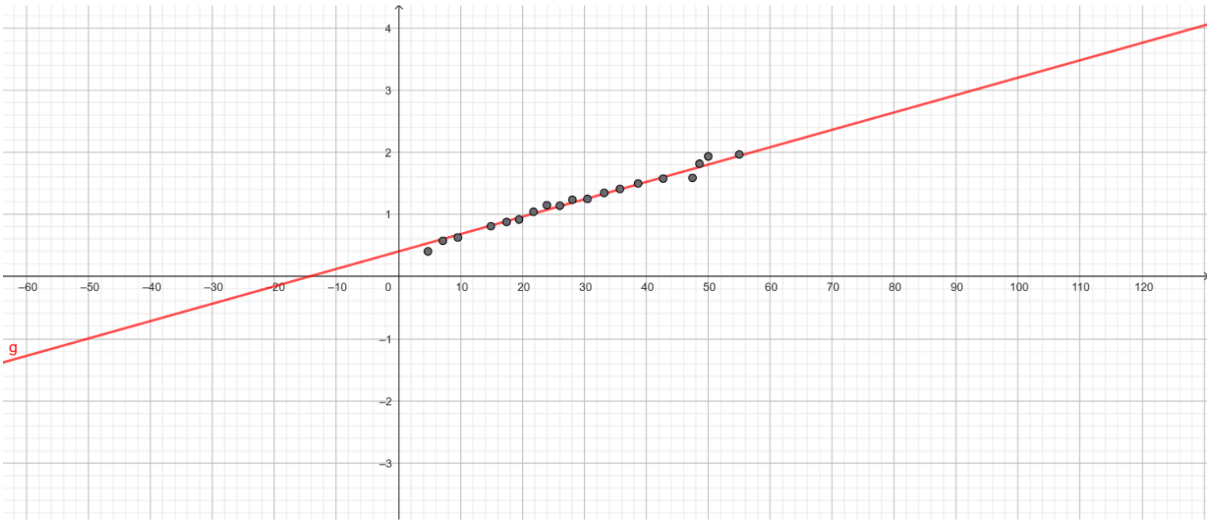
$$v = 0.028(\theta) + 0.404$$

This equation was derived from the manually drawn graph and represents the relationship between ramp angle and average speed.

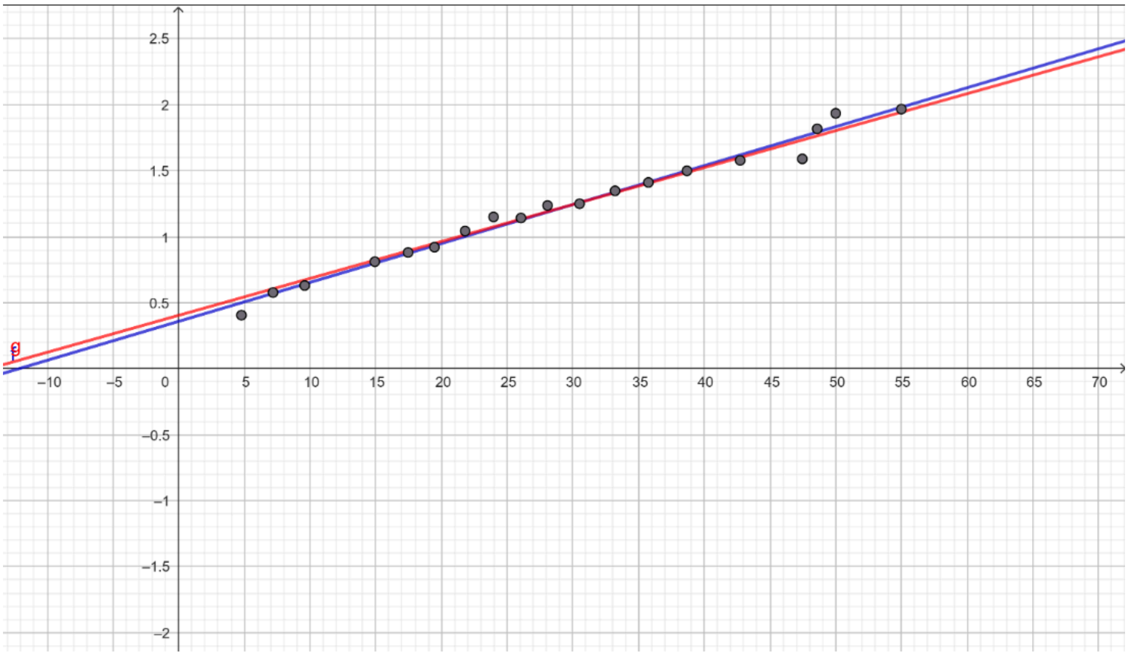


2.4 Using Desmos to Confirm the Model

A scatter plot of the same data was plotted on Desmos, and a linear regression line was fitted. The equation was $v = 0.028(\theta) + 0.40$.



This was the equation (0.028x + 0.36) fitted into Desmos to get this line.



To verify the manually created model, the experimental data was plotted in Desmos. Then, the specific line derived from the hand-drawn graph, v = 0.028(θ) + 0.404, was plotted on the same axes. It was observed that this manually calculated line is an excellent fit for the data points. When compared to the official regression line generated by Desmos (v=0.028θ+0.40), the lines were nearly identical. It was further observed that both lines share the same gradient (0.028). This confirms that the calculated rate of change (speed increase per degree) is accurate. The minor difference in their y-intercepts (0.404 vs 0.40) is negligible, confirming the manual model is a strong representation of the data's trend.

Regression Model

A linear regression model was found to present the relationship between ramp angle and average speed:

$$Average\ Speed = 0.03 \cdot \theta + 0.36$$

Prediction

To find the required angle θ for the machine's target speed of 1.63 m/s, we substitute:

Comparison:

- Manual line $v = 0.028\theta + 0.404$

Domain and Range:

- Domain: $5.2° \leq \theta \leq 18.4°$ (based on experimental range)
- Range: 0.55 m/s ≤ v ≤ 0.92 m/s

The model should not be used outside this domain, as the relationship may become nonlinear due to physical factors like air resistance at steeper angles

2.5 Strengths and Limitations of the Model

The mathematical model developed in this investigation shows several strengths that improved its reliability and usefulness. The strong linear relationship observed between the angle of elevation and the average speed is well supported by the data recorded. This is evident from the scatter plot, where all the data points align very closely with the line of best fit. The model allows a simple prediction of speed based on the angle, which makes it highly practical for real world applications such as the calibration of a tennis machine. Moreover, conducting three repeated trials for each angle reduced the impact of random errors in timings. This further increases the accuracy and reliability of the average speed measurements.

Additionally, the experiment incorporated mathematical precision by using trigonometry to accurately convert vertical height into angle. The purpose of this is to make sure the angle values were not assumed but deprived through valid mathematical reasoning. Despite these strengths, the model has several limitations that are important to be considered. It is based on idealised conditioning, assuming uniform acceleration, and not including additional factors such as air resistance, rolling imperfections and the surface of the ramp. These physical variables can cause minor inconsistencies between the predicted and actual speeds in the real-world setting.

2.6 Using the Model to Solve the Problem

The goal was to find the ramp angle required for a target speed of **1.63 m/s**. Substituting into the model:

$$1.63 = 0.028(\theta) + 0.404 \therefore \theta = \frac{1.63-0.404}{0.028} = 43.79°$$

Finding the required height:

$$h = 120 \times \sin sin\,(43.79°) \approx 120 \times 0.693 \approx 83.4\ cm$$

Thus, the ramp must be set to approximately $43.8°$ with a vertical height of **83 cm** to achieve the target speed.

2.7 Reasonableness of the solution

The developed model (v=0.028(θ)+0.404) is good and effective within its tested domain (5°-18°). Its strength lies in the highly linear data, which makes a straight-line equation a very reliable fit. However, the model is limited by its assumptions. Their gradients are identical, and their y-intercepts differ by only 0.004. This high degree of similarity verifies that the manual calculations are accurate and that the linear model is a good fit for the data. A further observation from the model is that for every one-degree increase in the ramp's angle, the model predicts the speed of the ball will increase by approximately 0.028 m/s.

3.0 Conclusion

This investigation successfully developed a mathematical model to describe the relationship between the ramp angle and the speed of a tennis ball rolling down it. By conducting an experiment, collecting data, calculating angles using trigonometry and deriving a line of best fit both manually and using Desmos, the equation:

$$v = 0.028(\theta) + 0.404$$

Was obtained. This equation shows that for every one degree increase in ramp angle, the speed of the ball increases by approximately by 0.028 m/s. Using this model, the required ramp angle to achieve the target speed of 1.63 m/s was determined to be $43.8°$, corresponding to a vertical height of 83 cm for a 120 cm ramp. The model is highly reliable within the tested range and provides a reasonable estimate outside it. Therefore, while the model itself is a good fit for the collected data, the solution it provides is mathematically correct but likely not accurate in a real-world scenario due to the limitations of its underlying assumptions.